Przegląd Solny / Salt Review, 2017, **13:** 66–74

A quasi-stationary model of salt leaching

Quasi-stacjonarny model ługowania soli

Jerzy Rybka, Kazimierz Urbańczyk

Ośrodek Badawczo-Rozwojowy Górnictwa Surowców Chemicznych „CHEMKOP" Sp. z o.o.,
ul. Wybickiego 7, 31-261 Kraków, jerzy.rybka@gmail.com; kazeku@poczta.onet.pl

Abstract

In classical models of the cavern leaching process, brine concentration is determined by calculating the increase (or decrease) of concentration in successive time-steps, based on the salt balance. In a quasi-stationary model it is assumed that changes in brine concentration are negligible in the balance. Consequently, a concentration needs to be found at which the leaching rate allows temporary concentration stability to be achieved.

Here, both types of models are compared, using the simplest case possible, namely the beginning of initial cut leaching by the method of direct water and brine circulation with an isolated cavern roof.

In the case of cylindrical shape of the cavern, both models are compatible. The quasi-stationary model does not take into account the leaching history, as it determines its variable asymptote rather than the brine concentration itself. In the case considered, the formula of the quasi-stationary model is so simple that calculations can be made using an MS Excel spreadsheet. However, in the general case, the algorithm of the quasi-stationary model is much more complicated than that of the classical model, especially if the cavern shape is complex and the method of reverse circulation is applied during the cavern leaching process.

Key words: leaching process modelling, solution mining, rock salt

Streszczenie

W klasycznych modelach procesu ługowania kawern stężenie solanki wyznacza się przez obliczenie jego przyrostu (lub spadku) w kolejnych krokach czasowych na podstawie bilansu soli. W quasi-stacjonarnym modelu zakłada się, że przyrost stężenia jest pomijalny i wyznacza się takie stężenie, przy którym szybkość ługowania zapewnia chwilową stacjonarność stężenia.

Oba modele są tu porównane na najprostszym przypadku – początku ługowania wrębu, w prawym obiegu wody i solanki oraz z izolacją stropu. Oba modele są zgodne, dopóki kształt kawerny jest walcem. Model quasi-stacjonarny nie bierze pod uwagę historii ługowania i nie tyle wyznacza stężenie, ile jego zmienną asymptotę. W rozpatrywanym przypadku formuła quasi-stacjonarnego modelu jest tak prosta, że obliczenia można wykonać w arkuszu MS Excela. W ogólnym przypadku algorytm modelu jest dużo bardziej skomplikowany niż w modelach klasycznych, zwłaszcza gdy kształt kawerny nie jest prosty i stosuje się obieg lewy.

Słowa kluczowe: modelowanie procesu ługowania, eksploatacja otworowa, sól kamienna

Introduction

Computer simulation of the leaching process is presently the standard way of designing new caverns. The basic elements of the respective computer model include various salt balances in each cavern zone which constitute the basis for determining the distribution of salt concentration in the cavern brine. In classical models, the concentration changes in subsequent time steps result from salt balances (Russo 1981, Saberian 1984, Kunstman, Urbańczyk 1990, 1994, 2000). Based on the initial concentration distribution, the leaching rate and the displacement of cavern walls are determined. Then, using the transport equation and salt balances, the distribution of brine concentration is determined at the end of a given time step, and that result is adopted as the initial brine distribution parameter for a subsequent time step.

It is possible to model this process in a different manner, namely by assuming that changes in brine concentration at each time step can be neglected. The remaining terms of the balance equation create stationary condition, brine concentration can then be calculated.

The model which assumes no change in brine concentration during a time step is a quasi-stationary model. In this paper, results of such a quasi-stationary model will be compared with those of the classical model, using as an example one of the simplest cases: the beginning of the initial cut leaching by direct water and brine circulation, with an isolated cavern roof.

The quasi-stationary model in the simplest case

To illustrate the operation of the quasi-stationary model against that of the classical model, the simplest case of the salt balance will be used, namely that of initial cut leaching by direct water and brine circulation, with an isolated cavern roof.

The cavern shape is cylindrical, the roof is isolated and is not relocated, the side walls are leached, and the cavern bottom becomes covered by insoluble material and rises, reducing the height of the cavern. It is assumed that brine in the cavern is continuously mixing and that its concentration inside the cavern is uniform.

Such a situation is illustrated in Fig.1.

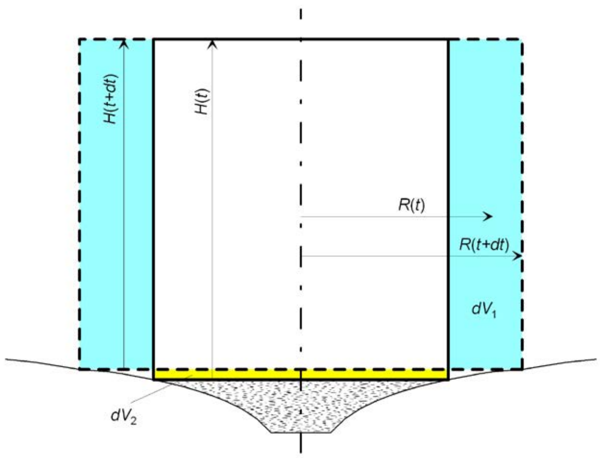

***Fig. 1.** Diagram of cavern development in a single time step at the initial phase of initial cut leaching.*

Ryc. 1. Schemat rozwoju kawerny w pojedynczym kroku czasowym w początkowej fazie ługowania wrębu.

In the classical model, the salt balance equation which defines the brine concentration compares the amount of salt found in the cavern at the end of a time step to the amount of salt found at the beginning of this time step, increased by the amount of leached salt and reduced by the salt extracted from the cavern and by salt remaining in the brine, in the insoluble parts that have settled at the bottom of the cavern (in the sump). The salt balance equation then has the following form:

$$V(t+dt)C(t+dt) = V(t)C(t) + dV_1\,\rho(1-p_n) - dV_2\,n\,C(t') - Q\,C(t')dt \tag{1}$$

Where:

C – brine concentration [kg/m^3]

n – sump porosity

p_n – insoluble content of the rock salt [-]

Q – brine production rate in the cavern [m^3/s]

t – time [s]

t' – a moment within a time interval <t, $t+dt$>, for a typical explicit scheme $t' = t$

V – volume [m^3]

dV_1 – cavern volume increase, due to leaching [m^3]

dV_2 – cavern volume decrease, due to insoluble material being deposited at the bottom of the cavern [m^3]

ρ – salt rock density [kg/m^3]

$$V(t+dt) = V(t) + dV_1 - dV_2 \tag{2}$$

$$C(t+dt) = C(t) + dC \tag{3}$$

Integrating expressions (2) and (3) within expression (1), and neglecting second-order terms, one arrives at:

$$\begin{aligned} &V(t)C(t) + C(t)dV_1 - C(t)dV_2 + V(t)dC = \\ &= V(t)C(t) + dV_1\,\rho(1-p_n) - dV_2\,n\,C(t) - Q\,C(t)dt \end{aligned} \tag{4}$$

It is further assumed that all time-dependent variables have been taken at time t, unless a different time is explicitly mentioned.

The sump volume is related to the newly leached volume in proportion to the insoluble matter content in rock salt.

$$(1-n)dV_2 = p_n\,dV_1 \tag{5}$$

The salt balance can now be expressed as follows:

$$dC = \left(\frac{\rho}{C} - 1\right)(1-p_N)C\frac{dV_1}{V} - \frac{QC}{V}dt \tag{6}$$

In the case under consideration, the following expressions can be applied:

$$V = \pi R^2 H \tag{7}$$

$$dV_1 = 2\pi R H k \frac{(C_N - C)^{1.5} C_N^{\;0,5}}{C_{N,20}^{\;2}} dt \tag{8}$$

Where:

R – cylindrical cavern radius [m]

H – cavern height [m] between the isolated roof and the rising sump

k – leaching rate coefficient [m/s]

C_N – saturation value of brine concentration [kg/m^3]

$C_{N,20}$ – saturation value of concentration at the temperature of 20°C

Finally:

$$dC = \left(\frac{\rho}{C} - 1\right)(1 - p_N)\frac{2}{R}k\frac{(C_N - C)^{1.5} C_N^{0,5}}{C_{N,20}^{2}} C\,dt - \frac{QC}{\pi R^2 H}dt \qquad (9)$$

The change of brine concentration during a single time step can now be calculated using the above formula (9) which describes the manner in which the classical model operates.

The brine concentration increase given by equation (9) can be a small difference between two much larger quantities, thus being prone to disturbances. In the quasi-stationary model, we assume that dC can be neglected and thus the balance equation becomes the stationary condition:

$$\left(\frac{\rho}{C} - 1\right)(1 - p_N)\frac{2}{R}k\frac{(C_N - C)^{1.5} C_N^{0,5}}{C_{N,20}^{2}} - \frac{Q}{\pi R^2 H} = 0 \qquad (10)$$

The brine concentration is derived from the leaching rate and can be iterated as follows:

$$C_{i+1} = \left(\frac{1}{C_N} + \left(\frac{Q}{2\pi HRk(1 - p_N)(\rho - C_i)}\right)^{2/3} C_i^{1/3}\left(\frac{C_{N,20}}{C_N}\right)^{4/3}\right)^{-1} \qquad (11)$$

Formula (11) is simple enough to be evaluated using a MS Excel spreadsheet.

Generally, in a quasi-stationary model, the time step is realised as follows. Based on the cavern shape and the injection rate, a stationary condition is obtained. Thus, a leaching rate must be selected which satisfies that stationary condition. Consequently, the brine concentration is needed which will give the appropriate leaching rate. Once that leaching rate is obtained, a new cavern shape can be determined at the end of the given time step and a new stationary condition is obtained (cf. Fig. 2.).

Comparison between quasi-stationary and classical models

To compare results of quasi-stationary and classical models, the following case has been considered:

Initial height of the leached interval	20 m
Initial radius	0.2 m
Insoluble part content	3%
Porosity of the sump section,	33%
corresponding to the loosening factor	1.5
Horizontal leaching rate at 20°C	10.135 mm/h
(with fresh water)	$= 2.815 \times 10^{-6}$ m/s
Cavern temperature	19.5°C
Horizontal leaching rate at 19.5°C	10.00 mm/h
	$= 2.778 \times 10^{-6}$ m/s
Brine concentration at 19.5°C	320 kg/m^3
Limiting dissolution angle	15°
Rock salt density	2,155 kg/m^3

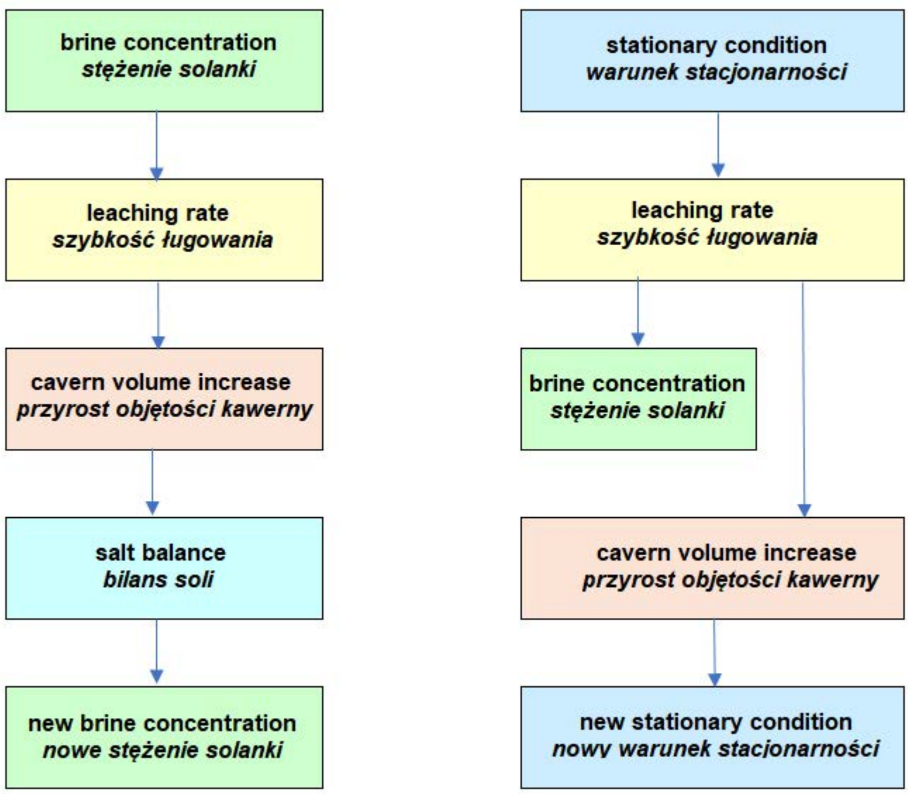

Fig. 2. *Time-step diagrams in a classical model (left) and a quasi-stationary model (right).*
Ryc. 2. Schematy kroków czasowych w modelu klasycznym (po lewej) i quasi-stacjonarnym (po prawej).

Quasi-stationary modelling began with a one-minute time step. Subsequent timing of steps was determined depending on the changes of concentration:

$$dt_{i+1} = dt_i \frac{0,1}{|C_i - C_{i-1}|}$$

Additionally, two limits were introduced:
- time step limit: $dt \leq 5$ days
- limit connected with the limiting inclination angle:

$$\frac{H_i - H_{i+1}}{R_{i+1} - R_i} \geq \mathrm{tg}15°$$

Classical modelling was carried out using the WinUbro-Net software, with a depth approximation step of 0.2 m and a time step limit of $dt \leq 1$ hour.

In the comparison of brine concentration dependences (Fig. 3.), one may note discrepancies between results of the two models – over the initial and late leaching times. Over the initial leaching times these are due to differences in the initial brine concentrations applied in each model. In the WinUbro model, the initial brine concentration was 290 kg/m^3, while in the quasi-stationary model it was 11-30 kg/m^3, depending on the brine production rate, at H=20 m and R=0,2 m. However, this discrepancy quickly disappears, as the volume of the initial borehole is small, and its effect on brine concentration is only temporary.

At extended leaching times, above some 50 days, the discrepancy between model results rises systematically with time. This is caused by the cylindrical cavern shape adopted in the quasi-stationary model. As the sump inclination reaches the limiting dissolution angle, the sump level rises more rapidly than due to the amount of insoluble material, resulting in the cavern volume calculation becoming too low in the quasi--stationary model. This is clearly seen in the cavern volume development graph (Fig. 4.). It also suggests that the conical zone of the cavern, neglected in steps (7) and (11), should be taken into account.

Other features of the quasi-stationary model become apparent as the injection rate is varied (Fig. 5.).

In the results of the quasi-stationary model, brine concentration rises immediately after a new injection, while in the actual leaching process its rise is more gradual. This shows that the quasi-stationary model determines the asymptote of brine concentration rather than its actual course - leaching history is not taken into account in the quasi-stationary model. In fact, every large change of the injection rate was followed

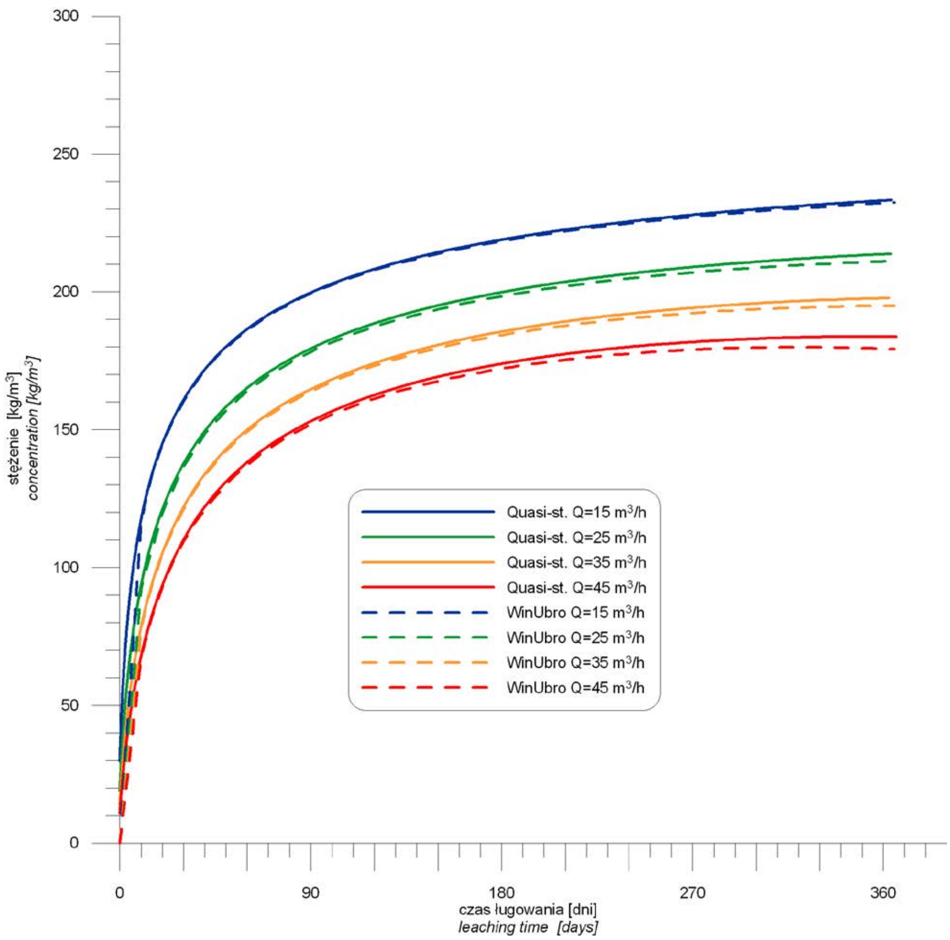

Fig. 3. *Comparison of quasi-stationary and classical models: brine concentration.*
Ryc. 3. Porównanie modeli quasi-stacjonarnego i klasycznego: stężenie solanki.

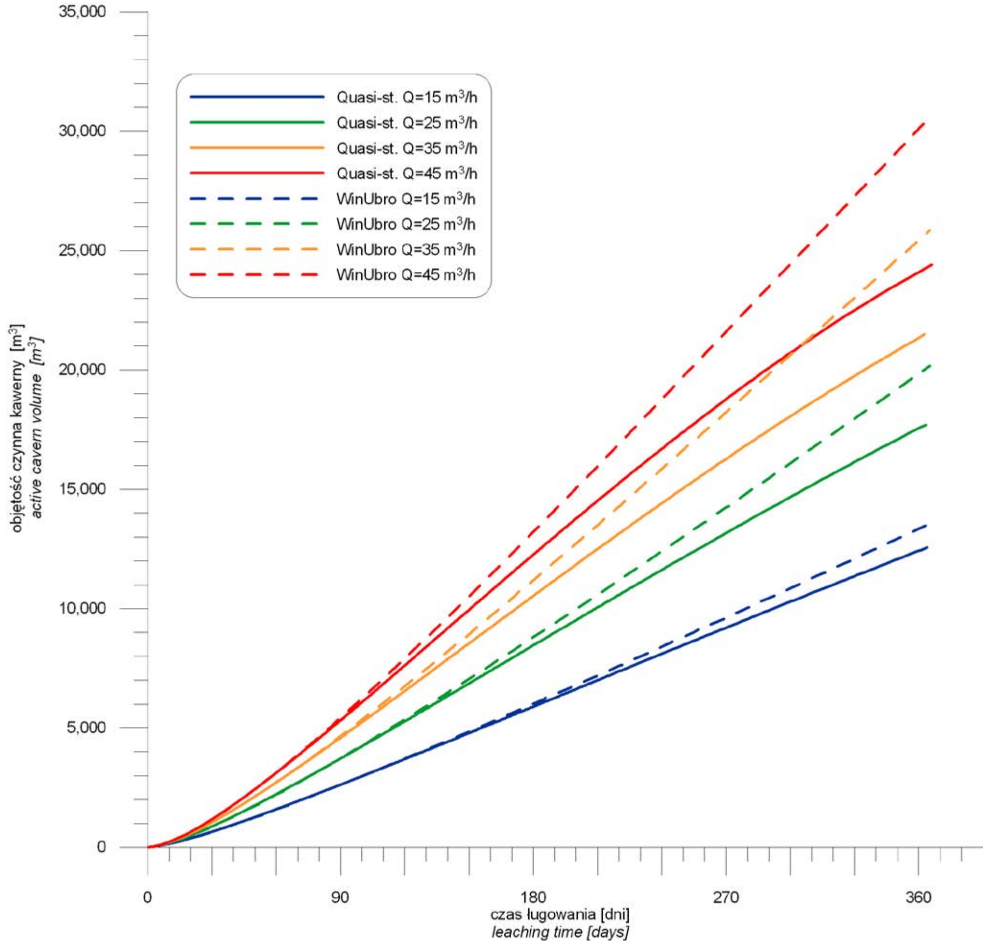

Fig. 4. *Comparison of quasi-stationary and classical models: cavern volume development.*
Ryc. 4. Porównanie modeli quasi-stacjonarnego i klasycznego: rozwój objętości kawerny.

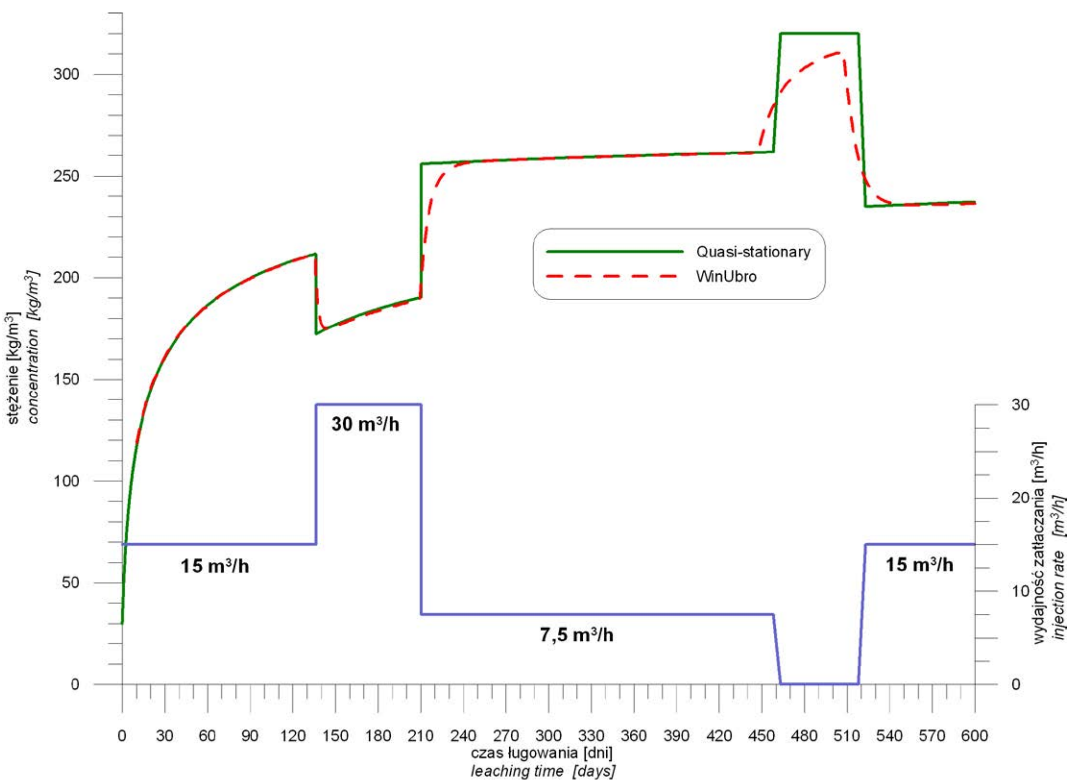

Fig. 5. *Comparison of quasi-stationary and classical models: brine concentration at varied injection rates.*
Ryc. 5. Porównanie modeli quasi-stacjonarnego i klasycznego: – stężenie solanki przy zmiennym zatłaczaniu.

by a period of process instability, which required the introduction of correction factors over these non-stationary periods.

A quasi-stationary model in the general case

The problems observed within the quasi-stationary model concerning non-stationary periods and non-cylindrical cavern shapes could be resolved rather easily. Quasi-stationary models can also be built for more complicated cases, e.g. if the internal tubing shoe is several meters above the sump, or for the zone below the internal tubing shoe or cavern slice between the tubing shoes in reverse circulation, etc. Equations similar to equations (10) and (11) can be readily derived for such cases.

However, in the general case, the above-described quasi-stationary model, could work in a manner similar to the Sansmic (Russo 1981, Kunstman, Urbańczyk 2000) or Salgas (Saberian 1984, Kunstman, Urbańczyk 2000) models where the approximation of the cavern contour is closely connected with the division of the cavern into cells (slices). It is an open problem, however, how to combine the quasi-stationary algorithm with cavern wall displacement independently of division of cavern volume into cells, as implemented in WinUbro (Kunstman, Urbańczyk 1990, 1994). Difficulties arise especially between the tubing shoes in reverse circulation, where wall inclination changes, or where the "old flat roof" moves upward from one zone to another.

Such problems could possibly be solved by applying iteration routines. Yet iterations are also required to derive brine concentration from the stationary condition. Thus, even if such iterations were implemented, the quasi-stationary model calculation would be much slower than the classical one.

On the other hand, the quasi-stationary model should be less prone to numerical disturbances than the classical model, where changes in concentration in the balance equation are numerically small differences between two much larger quantities. Fortunately, it is the negative feedback that keeps the classical model close to equilibrium, because any increase of brine concentration causes a decrease of the leaching rate. At most, in some cases, poor accuracy of numeric calculations may lead to some fluctuations. The quasi-stationary model is expected to be more stable

It is difficult to conclude whether the quasi-stationary model would find any future practical application or whether it will remain a subject of purely theoretical interest.

Summary

The quasi-stationary model uses the same equations as the classical models, but in a different manner. In the former case, an additional assumption is made that brine concentration changes very slowly and thus the concentration change can be neglected in the salt balance equation, which thus turns into a stationary condition. Then, a leaching rate is found which satisfies this condition, and next the brine concentration leading to this leaching rate. The specific leaching rate produces a new cavern shape and a new stationary condition is reached.

The simplest case, the beginning of initial cut leaching by direct water and brine circulation, with isolated cavern roof, was selected to compare results of the quasi-stationary model with those of the classical model (WinUbroNet). The results from either model were quite similar for that case. However, differences arose at the beginning of the leaching process which quickly disappeared, caused by the high brine concentration in the borehole before the start of the leaching operation. The quasi-stationary model did not take into account the leaching history. Also differences were apparent at later stages, after some tens of days, increasing with time, due to the shape of the cavern not remaining cylindrical at those stages.

Transient non-stationary periods, handled differently by these models, also appeared at times at which the injection rate was significantly changed.

One could derive a quasi-stationary model for more complex cases and cavern shapes and find corrections to handle the non-stationary periods. However, an open issue is how to combine the quasi-stationary algorithm with cavern wall displacement independently of division of cavern volume into cells. Iterative routines could be a possible solution, in addition to those required to derive brine concentration from the stationary condition. Thus, even if implemented in such iterative manner, the quasi-stationary model would be much slower than the classical model. On the other hand, the stability of the quasi-stationary model is expected to be higher.

It is difficult to conclude whether the quasi-stationary model would find any future practical application or whether it will remain a subject of purely theoretical interest.

Podsumowanie

Obecnie standardem jest projektowanie nowych modeli w oparciu o symulację komputerową. Jednym z podstawowych elementów komputerowego modelu ługowania kawerny są bilanse soli w różnych strefach kawerny, na których podstawie wyznacza się rozkład stężenia solanki w kawernie. Na podstawie początkowego rozkładu stężenia wyznacza się szybkość ługowania i postęp ociosu kawerny. Następnie na podstawie równań transportu i bilansu soli wyznacza się rozkład stężenia na koniec kroku czasowego czyli na początek następnego kroku czasowego.

Można skonstruować model inaczej, zakładając, że zmiana stężenia w kroku czasowym jest do pominięcia i z pozostałych wyrazów bilansu odwikłać stężenie. Taki model, w którym w każdym kroku przyjmuje się stałość stężenia, jest modelem quasi-stacjonarnym.

Dla zilustrowania tej metody zostanie tu użyty najprostszy przypadek bilansu soli – w początkowej fazie ługowania wrębu obiegiem prawym, z izolacją stropu.

Kawerna jest walcem, jej strop jest izolowany i nie przemieszcza się, jej ocios jest rozługowywany, jej spąg ulega zasypywaniu, skracając przez to wysokość ociosu.

Zakłada się, że solanka w kawernie ulega nieustannemu mieszaniu i stężenie w niej jest w każdym miejscu jednakowe. Sytuację przedstawia Ryc. 1.

Równanie bilansu, opisujące zmianę stężenia w pojedynczym kroku czasowym ma postać:

$$dC = \left(\frac{\rho}{C} - 1\right)\left(1 - p_N\right) \frac{2}{R} k \frac{\left(C_N - C\right)^{1.5} C_N^{\ 0,5}}{C_{N,20}^{\ 2}} C\,dt - \frac{QC}{\pi R^2 H} dt \qquad (9)$$

gdzie:

C – stężenie [kg/m³]

C_N – stężenie nasycenia [kg/m³]

$C_{N,20}$ – stężenie nasycenia w temperaturze 20°C

H – wysokość kawerny [m] pomiędzy izolowanym stropem a narastającym zasypem

k – współczynnik szybkości ługowania [m/s]

n – porowatość rząpia (zasypu)

p_n – udział części nierozpuszczalnych w skale solnej [-]

Q – wydajność produkcji solanki z kawerny [m³/s]

R – promień walcowej kawerny [m]

t – czas [s]

ρ – gęstość skały solnej [kg/m³]

W modelu quasi-stacjonarnym przyjmuje się, że dC jest do zaniedbania i stężenie odwikłuje się z równania:

$$\left(\frac{\rho}{C} - 1\right)\left(1 - p_N\right)\frac{2}{R} k \frac{\left(C_N - C\right)^{1.5} C_N^{\ 0,5}}{C_{N,20}^{\ 2}} - \frac{Q}{\pi R^2 H} = 0 \qquad (10)$$

Zdaniem autorów, najlepiej jest iterować następująco:

$$C_{i+1} = \left(\frac{1}{C_N} + \left(\frac{Q}{2\pi HRk\left(1 - p_N\right)\left(\rho - C_i\right)}\right)^{2/3} C_i^{\ 1/3} \left(\frac{C_{N,30}}{C_N}\right)^{4/3}\right)^{-1} \qquad (11)$$

Mając początkowy kształt kawerny i wydajność, konstruuje się warunek stacjonarności i wyznacza stężenie, przy którym warunek ten jest spełniony. Z tego wynika szybkość ługowania i zmiana kształtu kawerny w kroku czasowym oraz nowy warunek stacjonarności po kroku czasowym (por. Ryc. 2).

Dla celów porównania rozpatrzono następujący przypadek:

wysokość początkowa wrębu	20 m
początkowy promień	0,2 m
udział części nierozpuszczalnych	3%
porowatość części osiadających w rząpiu	33%
co odpowiada współczynnikowi rozluźnienia	1,5
szybkość ługowania bocznego przy 20°C	10,135 mm/h
(dla wody czystej)	= 2,815×10⁻⁶ m/s
temperatura w kawernie	19,5°C
szybkość ługowania bocznego przy 19,5°C	10,00 mm/h
	= 2,778×10⁻⁶ m/s
stężenie nasycenia w temperaturze 19,5°C	320 kg/m³.
graniczny kąt ługowania	15°
gęstość skały solnej	2.155 kg/m³

Modelowanie quasi-stacjonarne rozpoczęto krokiem czasowym 1 min., a dalsze kroki wyznaczano w zależności od zmian stężenia:

$$dt_{i+1} = dt_i \frac{0,1}{\left|C_i - C_{i-1}\right|}$$

jednak wprowadzając ograniczenie: $dt \leq$ 5 dni

oraz dodatkowe ograniczenie związane z kątem granicznym:

$$\frac{H_i - H_{i+1}}{R_{i+1} - R_i} \geq \mathrm{tg}15°$$

Modelowanie klasyczne wykonano programem WinUbroNet, stosując aproksymację głębokości krokiem 0,2 m i ograniczenie kroku czasowego $dt \leq$ 1 godz

Wyniki przedstawiają Ryc. 3 i Ryc. 4. Widać dwie różnice w wynikach. Pierwsza, na początku, wynikająca z innego stężenia początkowego – dla WinUbro 290 kg/m³, dla modelu quasi-stacjonarnego 11-30 kg/m³, zależnie od wydajności (quasi-stacjonarne dla H=20 m i R=0,2 m. Druga różnica pojawia się po kilkudziesięciu dniach, gdy założenie, że kawerna ma kształt walca przestaje być słuszne.

Model quasi-stacjonarny, sformułowany powyżej, nie bierze pod uwagę historii ługowania i faktu, że po każdej zmianie wydajności, następują okresy niestacjonarne. Widać je na Ryc. 5., gdzie zamodelowano ługowanie wrębu ze zmienną wydajnością.

Problem z niecylindrycznością kształtu i okresami niestacjonarnymi można łatwo rozwiązać. Jednak otwartym problemem pozostaje, jak połączyć quasi-stacjonarny algorytm z przemieszczaniem ociosu kawerny niezależnym od podziału na strefy bilansowe. Zwłaszcza problemem jest zmiana kąta nachylenia ociosu i przechodzenie byłego stropu z jednej strefy do drugiej. Można próbować rozwiązać ten problem poprzez iteracje, ale taki program będzie liczył dosyć wolno, w porównaniu z klasycznym.

Model quasi-stacjonarny jest natomiast mniej podatny na numeryczne zakłócenia niż model klasyczny, w którym przyrost stężenia bywa małą różnicą dwóch dużych wielkości.

Obecnie trudno przewidzieć, czy model quasi-stacjonarny znajdzie praktyczne zastosowanie, czy pozostanie ciekawostką teoretyczną.

References / *Literatura*

KUNSTMAN A.S., URBAŃCZYK K.M., 1990: UBRO A Computer Model for Designing Salt Caverns Leaching Process Developed at CHEMKOP. *SMRI Meeting Paper*, Paris October 1990.

KUNSTMAN A.S., URBAŃCZYK K.M., 1994: The new UBRO version – leaching simulation code for asymmetrical caverns – result of applying to a Mogilno gas cavern. *SMRI Meeting Paper*, Hannover, September 1994: 451-486

KUNSTMAN A.S., URBAŃCZYK K.M., 2000: Computer Models of the Salt Cavern Leaching Process – Evolution Over the Last 35 Years. *8th World Salt Symposium*, V. 1, p. 279, Elsevier Ed.

RUSSO A.J., 1981: A solution mining code for studying axisymmetric salt cavern formation. *Sandia Report* SAND81-1231: 1-32.

SABERIAN A., 1984: SALGAS User's Manual, Volume 1 – Theories, Formulas & Program Description. *SMRI Research Project Report* No. 84-003-S.